## Transition Matrices

1. Construct a transition matrix that can be used to represent each of these diagrams.
a

b

2. 

A car rental firm has two branches: one in Bendigo and one in Colac. Cars are usually rented and returned to the same town. However, a small percentage of cars rented in Bendigo each week are returned in Colac, and vice versa. The diagram below describes what happens on a weekly basis. In one particular week, the depot in Colac has 50 cars and there are 40 cars in Bendigo.

A. Write the initial state matrix for this situation.
B. What is the transition matrix?
C. After one week how many cars will be returned to Bendigo?
D. After three weeks how many cars will be returned to Bendigo?
E. In the long run how many cars are at each branch of the car rental firm?
3. A car insurance company finds that:

- $22 \%$ of car drivers involved in an accident this year (A) are also expected to be involved in an accident next year
- $9 \%$ of drivers who are not involved in an accident this year $(\mathrm{N})$ are expected to be involved in an accident next year. Use this information to complete this transition diagram then write a transition matrix.


4. Two fast-food outlets, Jill's (J) and Pete's (P), are located in a small town. The transition matrix provided describes how people in this town move between these two outlets on a weekly basis.
A. Construct a transition matrix to describe this situation.
B. Initially, 400 people eat at Jill's and 400 eat at Pete's. Write down a column matrix $S_{0}$ that describes this
 situation.
C. How many of these people do we expect to go to Jill's the next week? How many to Pete's?
D. How many do we expect to go to Jill's after 5 weeks? How many to Pete's?
E. In the long term, how many do we expect to eat at Jill's every week? How many of these people do we expect to eat at Pete's?
5. 

The Whiteoak Theatre Club has 200 members who buy tickets for every concert. The members can choose seats from three different classes, A, B or C. For each concert, the choice of seat class for these members can be determined using the transition matrix T , shown below:

## this concert


A. Complete the transition diagram above by adding all the missing information.
B. The number of seats in each class chosen by these members for the final concert this year is shown in matrix $\mathrm{S}_{0}$. What percentage of these members chose A class seats for the final concert this year?

$$
S_{0}=\left[\begin{array}{l}
16 \\
96 \\
88
\end{array}\right] \begin{aligned}
& A \\
& B \\
& C
\end{aligned}
$$

C. For the first concert next year, some members will choose a different seat class from the seat class that they chose for the final concert this year.
What percentage of the 200 members are expected to change from $B$ class seats at the final concert this year to A class seats for the first concert next year?
D. Write down the state matrix, $S_{1}$, for the expected number of members and their choice of seat class for the first concert next year. Write your answer correct to one decimal place.
E. In the long term, how many members would be expected to buy B class seats for a concert?

