Cambridge Senior General Mathematics AC/VCE Units 1 & 2
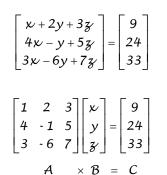Chapter 5 Matrices: **Skillsheet 5K**

Solve to find x, y and z

$$x + 2y + 3z = 9$$
$$4x - y + 5z = 24$$
$$3x - 6y + 7z = 33$$

1. The three simultaneous equations can be represented by the matrix equation shown.

$$\begin{bmatrix} x + 2y + 3z \\ 4x - y + 5z \\ 3x - 6y + 7z \end{bmatrix} = \begin{bmatrix} 9 \\ 24 \\ 33 \end{bmatrix}$$

2. The left-hand side of the matrix equation in step 1 can be written as the product of two matrices.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & -1 & 5 \\ 3 & -6 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 24 \\ 33 \end{bmatrix}$$

$$A \quad \times \quad B \quad = \quad C$$

3. Name the matrices as shown. Matrix B contains the solutions to the simultaneous equations.

4. Enter matrix A and matrix C.

If you need help, see "How to enter a matrix" pages 220 and 221 in the print textbook.

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & -1 & 5 \\ 3 & -6 & 7 \end{bmatrix} \to a \qquad \begin{bmatrix} 1 & 2 & 3 \\ 4 & -1 & 5 \\ 3 & -6 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 9 \\ 24 \\ 33 \end{bmatrix} \to c \qquad \begin{bmatrix} 9 \\ 24 \\ 33 \end{bmatrix}$$

$$B = A^{-1} \, C$$

5. We want to find the values of matrix B

Since
$$A \times B = C$$
$$A^{-1} \times A \times B = A^{-1} \times C$$
$$I \times B = A^{-1} \times C$$
$$B = A^{-1} \times C$$

$$a^{-1} \cdot c \qquad \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

6. Write matrix B.

$$B = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

7. Write the solutions to the equations.

So $x = 2$, $y = -1$ and $z = 3$

8. Check the solutions by substituting the values of x, y and z into at least one of the original equations.

Using: $x + 2y + 3z = 9$
LHS $= 2 + 2(-1) + 3(3)$
$= 9$
$= RHS$, as required.

Exercise

Use matrix methods on your CAS calculator to solve the following simultaneous equations.

1 $2x + y + 5z = 8$

$3x - 4y + 6z = -13$

$9x + 7y - 8z = -15$

2 $5x - 2y + 6z = -26$

$4x - y + 9z = -4$

$8y + 11z = 89$ *

*Hint: In Question 2, consider the third equation as $0x + 8y + 11z = 89$

3 $4w + 3x - 5y + 6z = 50$

$2w - 7x + y + 8z = 26$

$9w - 10x - 4y + 11z = 64$

$5w + 2x + 8y - 7z = -40$