

# 1

## Computation and practical arithmetic

- ▶ How do we use a variety of mathematical operations in the correct order?
- ▶ How do we add, subtract, multiply and divide directed numbers?
- ▶ How do we find powers and roots of numbers?
- ▶ How do we round numbers to specific place values?
- ▶ How do we write numbers in standard form?
- ▶ What are and how do we use significant figures?
- ▶ How do we convert units of measurements?
- ▶ How do we express ratios in their simplest form?
- ▶ How do we solve practical problems involving ratios, percentages and the unitary method?
- ▶ How do we use and interpret log scales that represent quantities that range over multiple orders of magnitude?

## Introduction

This chapter revises basic methods of computation used in general mathematics. It will allow you to carry out the necessary numerical calculations for solving problems. We will begin with the fundamentals.

### 1A Order of operations

Adding, subtracting, multiplying, dividing and squaring are some examples of operations that are used in mathematics. When carrying out a sequence of arithmetic operations, it is necessary to observe a definite sequence of rules. These rules, defining the order of operations, have been devised and standardised to avoid confusion.

#### Order of operation

The rules are to:

- always complete the operations in brackets first
- then carry out the division and multiplication operations (in order, from left to right)
- then carry out the addition and subtraction operations (in order, from left to right).

These rules can also be remembered by using **BODMAS**.

- B** Brackets come first
- O** If a fraction **O**f a number is required or **O**rders (powers, square roots), you complete that next
- DM** Division and Multiplication, working left to right across the page
- AS** Addition and Subtraction, working left to right across the page

A calculator, with *algebraic logic*, will carry out calculations in the correct order of operations. However, particular care must be taken with brackets.

#### Pronumeral

A number or **pronumeral** (letter) placed in front of a bracket means that you multiply everything in the bracket by that number or pronumeral.

$$4(8) \text{ means } 4 \times 8 = 32$$

$$5(x - 9) = 5x - 45$$

$$a(3a + 6) = 3a^2 + 6a$$

**Example 1** Using correct order of operation

Evaluate the following.

**a**  $3 + 6 \times 8$

**b**  $(3 + 6) \times 8$

**c**  $8 \div 2 - 2$

**d**  $23 - (8 - 5)$

**e**  $(4)3 - 2$

**f**  $3 + 5(x - 1)$

**g**  $(3 \times 8.5 - 4) - (4.1 + 5.4 \div 2)$

**Solution**

**a**  $3 + 6 \times 8 = 3 + 48$   
 $= 51$

**b**  $(3 + 6) \times 8 = 9 \times 8$   
 $= 72$

**c**  $8 \div 2 - 2 = 4 - 2$   
 $= 2$

**d**  $23 - (8 - 5) = 23 - 3$   
 $= 20$

**e**  $(4)3 - 2 = 12 - 2$   
 $= 10$

**f**  $3 + 5(x - 1) = 3 + 5x - 5$   
 $= 5x - 2$

**g**  $(3 \times 8.5 - 4) - (4.1 + 5.4 \div 2) = (25.5 - 4) - (4.1 + 2.7)$   
 $= 21.5 - 6.8$   
 $= 14.7$

**Exercise 1A****Example 1a-d****1** Evaluate the following, without using a calculator.

**a**  $5 + 4 \times 8$

**b**  $4 \times 3 - 7$

**c**  $7 \times 6 - 4 + 4 \times 3$

**d**  $15 \div 3 + 2$

**e**  $3 + 12.6 \div 3$

**f**  $4 \times (8 + 4)$

**g**  $15 - 9 \div 2 + 4 \times (10 - 4)$

**h**  $(3.7 + 5.3) \div 2$

**i**  $8.6 - 3 \times 2 - 6 \div 3$

**j**  $(3 \times 4 - 3) \div (2 - 3 \times 4)$

**Example 1e****2** Use your calculator to find the answers to the following.

**a**  $(8.23 - 4.5) + (3.6 + 5.2)$

**b**  $(17 - 8.7) - (73 - 37.7)$

**c**  $(6.2 + 33.17) \times (6.9 - 6.1)$

**d**  $(3.2 + 0.5 \div 2.5) \div (8.6 - 1.3 \times 4)$

**Example 1f-g****3** Evaluate the following.

**a**  $9(3)$

**b**  $2(x - 7)$

**c**  $10(5 - y)$

**d**  $w(8 - 2)$

**e**  $k(k + 8)$

**f**  $27(2) - 3(8)$

**g**  $(5 - 3)x + 7(2)$

**h**  $3(5) \times 2 - 8$

**i**  $3(x + 1) - 8$

**j**  $4 - 2(x + 3)$



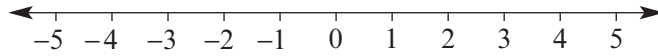


## 1B Directed numbers

Positive and negative numbers are *directed numbers* and can be shown on a number line.

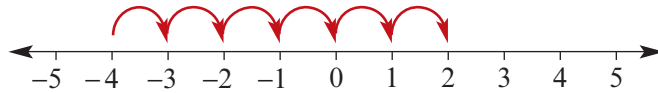
### ► Addition and subtraction

It is often useful to use a number line when adding directed numbers.



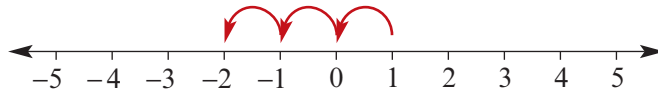
Adding a positive number means that you move to the right.

Example:  $-4 + 6 = 2$



Adding a negative number means that you move to the left.

Example:  $1 + (-3) = -2$



When subtracting directed numbers, you add its opposite.

Example:  $-2 - 3$  is the same as  $-2 + (-3) = -5$

Example:  $7 - (-9) = 7 + 9 = 16$

### ► Multiplication and division

Multiplying or dividing two numbers with the *same* sign gives a *positive* value.

Multiplying or dividing two numbers with *different* signs gives a *negative* value.

#### Multiplication and division with directed numbers

$$+ \times + = +$$

$$- \times - = +$$

$$+ \div + = +$$

$$- \div - = +$$

$$+ \times - = -$$

$$- \times + = -$$

$$+ \div - = -$$

$$- \div + = -$$

**Example 2** Using directed numbers

Evaluate the following.

**a**  $6 - 13$

**b**  $(-5) - 11$

**c**  $9 - (-7)$

**d**  $(-10) - (-9)$

**e**  $5 \times -3$

**f**  $(-8) \times (-7)$

**g**  $(-16) \div 4$

**h**  $(-60) \div (-5)$

**i**  $(-100) \div (-4) \div (-5)$

**j**  $(-3)^2$

**Solution**

**a**  $6 - 13 = 6 + (-13) = -7$

**b**  $(-5) - 11 = (-5) + (-11)$   
 $= -16$

**c**  $9 - (-7) = 9 + 7$   
 $= 16$

**d**  $(-10) - (-9) = (-10) + 9$   
 $= -1$

**e**  $5 \times -3 = -15$

**f**  $(-8) \times (-7) = 56$

**g**  $(-16) \div 4 = -4$

**h**  $(-60) \div (-5) = 12$

**i**  $(-100) \div (-4) \div (-5) = 25 \div (-5)$   
 $= -5$

**j**  $(-3)^2 = (-3) \times (-3)$   
 $= 9$

**Exercise 1B****Example 2a-d****1** Without using a calculator, find the answers to the following.

**a**  $6 - 7$

**b**  $-10 + 6$

**c**  $-13 + (-3)$

**d**  $-7 + 10$

**e**  $-7 - 19$

**f**  $(-18) - 7$

**g**  $(-9) - 3$

**h**  $4 - (-18)$

**i**  $18 - (-4)$

**j**  $15 - (-17)$

**k**  $16 - (-12)$

**l**  $(-3) - (-13)$

**m**  $(-12) - (-6)$

**n**  $(-21) - (-8)$

**Example 2e-j****2** Without using a calculator, find the answers to the following.

**a**  $(-6) \times 2$

**b**  $(-6)(-4)$

**c**  $(-10) \div (-4)$

**d**  $15 \div (-3)$

**e**  $(5 + 2) \times 6 - 6$

**f**  $-(-4) \times -3$

**g**  $-7(-2 + 3)$

**h**  $-4(-7 - (2)(4))$

**i**  $-(3 - 2)$

**j**  $-6 \times (-5 \times 2)$

**k**  $-6(-4 + 3)$

**l**  $-(-12 - 9) - 2$

**m**  $-4 - 3$

**n**  $-(-4 - 7(-6))$

**o**  $(-5)(-5) + (-3)(-3)$

**p**  $8^2 + 4(0.5)(8)(6)$

**1C Powers and roots****► Squares and square roots**When a number is multiplied by itself, we call this the *square* of the number.

$$4 \times 4 = 4^2 = 16$$

- 16 is called the *square* of 4 (or 4 squared).
- 4 is called the *square root* of 16.
- The square root of 16 can be written as  $\sqrt{16} = 4$ . ( $\sqrt{\quad}$  is the square root symbol)

### ► Cubes and cube roots

When a number is squared and then multiplied by itself again, we call this the *cube* of the number.

$$4 \times 4 \times 4 = 4^3 = 64$$

- 64 is called the *cube* of 4 (or 4 cubed).
- 4 is called the *cube root* of 64.
- The cube root of 64 can be written as  $\sqrt[3]{64} = 4$ . ( $\sqrt[3]{\phantom{x}}$  is the cube root symbol)

### ► Other powers

When a number is multiplied by itself a number of times, the values obtained are called *powers* of the original number.

For example,  $4 \times 4 \times 4 \times 4 \times 4 = 1024 = 4^5$ , which is read as ‘4 to the *power* of 5’.

- 4 is the fifth root of 1024.
- $\sqrt[5]{1024}$  means the fifth root of 1024.
- Another way of writing  $\sqrt{16}$  is  $16^{\frac{1}{2}}$ , which is read as ‘16 to the half’.
- Likewise,  $8^{\frac{1}{3}}$ , read as ‘8 to the third’, means  $\sqrt[3]{8} = 2$ .
- Powers and roots of numbers can be evaluated on the calculator by using the  $\wedge$  button.

#### Example 3 Finding the power or root of a number using a calculator

**a** Find  $8^3$ .

**b** Find  $8^{\frac{1}{3}}$ .

**Solution**

**a**

$8^3$	512
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**b**

$8^{(1/3)}$	2
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## Exercise 1C

**Example 3** 1 Find the value of the following.

**a**  $10^4$

**b**  $7^3$

**c**  $\sqrt{25}$

**d**  $\sqrt[3]{8}$

**e**  $2^6$

**f**  $12^4$

**g**  $9^{\frac{1}{2}}$

**h**  $169^{\frac{1}{2}}$

**i**  $1\,000\,000^{\frac{1}{2}}$

**j**  $64^{\frac{1}{3}}$

**k**  $32^{\frac{1}{5}}$

2 Find the value of the following.

**a**  $\sqrt{10^2 + 24^2}$

**b**  $\sqrt{39^2 - 36^2}$

**c**  $\sqrt{12^2 + 35^2}$

**d**  $\sqrt{(4+2)^2 - 11}$

**e**  $10(3+5) - (\sqrt{9} - 2)$

**f**  $\sqrt{(3+2)^2 - (5-2)^2}$



## 1D Approximations, decimal places and significant figures

Approximations occur when we are not able to give exact numerical values in mathematics. Some numbers are too long (e.g. 0.573 128 9 or 107 000 000 000) to work with and they are rounded to make calculations easier. Calculators are powerful tools and have made many tasks easier that previously took a considerable amount of time. Nevertheless, it is still important to understand the processes of rounding and estimation.

Some questions do not require an exact answer and a stated degree of accuracy is often sufficient. Some questions may only need an answer rounded to the nearest tenth, hundredth etc. Other questions may ask for an answer correct to two decimal places or to three significant figures.

### ► Rules for rounding

#### Rules for rounding

- 1 Look at the value of the digit to the right of the specified digit.
- 2 If the value is 5, 6, 7, 8 or 9, *round the digit up*.
- 3 If the value is 0, 1, 2, 3 or 4, *leave the digit unchanged*.

#### Example 4 Rounding to the nearest thousand

Round 34 867 to the nearest thousand.

#### Solution

- 1 Look at the first digit after the thousands. It is an 8.
- 2 As it is 5 or more, increase the digit to its left by one. So the 4 becomes a 5. The digits to the right all become zero. Write your answer.

**Note:** 34 867 is closer to 35 000 than 34 000

$$\begin{array}{r} \Downarrow \\ 34\mathbf{8}67 \\ 35\mathbf{0}00 \end{array}$$

### ► Scientific notation (standard form)

When we work with very large or very small numbers, we often use *scientific notation*, also called *standard form*.

To write a number in scientific notation we express it as a number between 1 and 10 multiplied by a power of 10.

**Scientific notation****Large numbers**

$$\begin{aligned} 249000000000 &= 2.49 \times 100\,000\,000\,000 \\ &= 2.49 \times 10^{11} \end{aligned}$$

The decimal point needs to be moved 11 places to the right to obtain the basic numeral.

Multiplying by  $10^{\text{positive power}}$  gives the effect of moving the decimal point to the right to make the number larger.

**Small numbers**

$$\begin{aligned} 0.000000002 &= 2.0 \div 1\,000\,000\,000 \\ &= 2.0 \times 10^{-9} \end{aligned}$$

The decimal point needs to be moved 9 places to the left to obtain the basic numeral.

Multiplying by  $10^{\text{negative power}}$  gives the effect of moving the decimal point to the left to make the number smaller.

**Example 5** Writing a number in scientific notation

Write the following numbers in scientific notation.

**a** 7 800 000

**b** 0.000 000 5

**Solution**

**a 1** Write 7 800 000 as a number between 1 and 10 (7.8) and decide what to multiply it by to make 7 800 000.

$$7\,800\,000 = 7.8 \times 1\,000\,000$$

*6 places*

$$7\,800\,000$$

**2** Count the number of places the decimal point needs to move and whether it is to the left or right.

*Decimal point needs to move 6 places to the right from 7.8 to make 7 800 000.*

**3** Write your answer.

$$7\,800\,000 = 7.8 \times 10^6$$

**b 1** Write 0.000 000 5 as a number between 1 and 10 (5.0) and decide what to divide it by to make 0.000 000 5

$$0.000\,000\,5 = 5.0 \div 10\,000\,000$$

*7 places*

$$0.000\,000\,5$$

**2** Count the number of places the decimal point needs to move and whether it is to the left or right.

*Decimal point needs to move 7 places to the left from 5.0 to make 0.000 000 5*

**3** Write your answer.

$$0.000\,000\,5 = 5.0 \times 10^{-7}$$



**Example 6** Writing a scientific notation number as a basic numeral

Write the following scientific notation numbers as basic numerals.

**a**  $3.576 \times 10^7$

**b**  $7.9 \times 10^{-5}$

**Solution**

**a 1** Multiplying 3.576 by  $10^7$  means that the decimal point needs to be moved 7 places to the right.

$$\begin{array}{l} 3.576 \times 10^7 \\ \quad \quad \quad \text{7 places} \\ \overbrace{3.5760000}^{\text{7 places}} \times 10^7 \\ = 35760000 \end{array}$$

**2** Move the decimal place 7 places to the right and write your answer. Zeroes will need to be added as placeholders.

**b 1** Multiplying 7.9 by  $10^{-5}$  means that the decimal point needs to be moved 5 places to the left.

$$\begin{array}{l} 7.9 \times 10^{-5} \\ \quad \quad \quad \text{5 places} \\ \overbrace{0.000079}^{\text{5 places}} \times 10^{-5} \\ = 0.000079 \end{array}$$

**2** Move the decimal place 5 places to the left and write your answer.

### ► Significant figures

The first non-zero digit, reading from left to right in a number, is the first *significant figure*. It is easy to think of significant figures as all non-zero figures, except where the zero is between non-zero figures. The number of significant figures is shown in red below.

For example:

Number	Significant figures	Explanation
596.36	5	All numbers provide useful information.
5000	1	We do not know anything for certain about the hundreds, tens or units places. The zeroes may be just placeholders or they may have been rounded off to give this value.
0.0057	2	Only the 5 and 7 tell us something. The other zeroes are placeholders.
0.00570	3	The last zero tells us that the measurement was made accurate to the last digit.
8.508	4	Any zeroes between significant digits are significant.
0.00906	3	Any zeroes between significant digits are significant.
560.0	4	The zero in the tenths place means that the measurement was made accurate to the tenths place. The first zero is between significant digits and is therefore significant.

**Rules for significant figures**

- 1** All non-zero digits are significant.
- 2** All zeroes between significant digits are significant.
- 3** After a decimal point, all zeroes to the right of non-zero digits are significant.

**Example 7** Rounding to a certain number of significant figures

Round 93.738 095 to:

- a** two significant figures      **b** one significant figure      **c** five significant figures

**Solution**

- a 1** Count the significant figures in 93.738 095 *There are eight significant figures.*
- 2** For two significant figures, start counting two non-zero numbers from the left. *93.738 095*
- 3** The next number (7) is 5 or more so we increase the previous number (3) by one (making it 4). Write your answer. *= 94 (two significant figures)*
- b 1** For one significant figure, count one non-zero number from the left. *93.738 095*
- 2** The next number (3) is less than 5 so we leave the previous number (9) as it is and replace the 3 with 0 to make only one significant figure. Write your answer. *= 90 (one significant figure)*
- c 1** For five significant figures, start counting five non-zero numbers from the left. *93.738 095*
- 2** The next number (0) is less than 5 so do not change the previous number (8). Write your answer. *= 93.738 (five significant figures)*



### Example 8 Rounding to a certain number of significant figures

Round 0.006 473 5 to:

- a** four significant figures    **b** three significant figures    **c** one significant figure

#### Solution

- a 1** Count the significant figures. *There are five significant figures.*
- 2** Count four non-zero numbers starting from the left. *0.0064735*
- 3** The next number (5) is 5 or more. Increase the previous number (3) by one (4). Write your answer. *= 0.006474 (four significant figures)*
- b 1** For three significant figures, count three non-zero numbers from the left. *0.0064735*
- 2** The next number (3) is less than 5 so we leave the previous number (7) as it is. Write your answer. *= 0.00647 (three significant figures)*
- c 1** For one significant figure, count one non-zero number from the left. *0.0064735*
- 2** The next number (4) is less than 5 so do not change the previous number (6). Write your answer. *= 0.006 (one significant figure)*

## ► Decimal places

23.798 is a decimal number with three digits after the decimal point. The first digit (7) after the decimal point is the first (or one) decimal place. Depending on the required accuracy we round to one decimal place, two decimal places, etc.

### Example 9 Rounding correct to a number of decimal places

Round 94.738 295 to:

- a** two decimal places    **b** one decimal place    **c** five decimal places

#### Solution

- a 1** For two decimal places, count two places after the decimal point and look at the next digit (8). *94.738295*
- 2** As 8 is 5 or more, increase the digit to the left of 8 by one. (3 becomes 4) Write your answer. *= 94.74 (to two decimal places)*

- b 1** For one decimal place, count one place after the decimal point and look at the next digit (3).  $94.\overline{738}295$
- 2** As 3 is less than 5, the digit to the left of 3 remains unchanged. Write your answer.  $= 94.7$  (to one decimal place)
- c 1** For five decimal places, count five places after the decimal point and look at the next digit (5).  $94.\overline{738}29\overline{5}$
- 2** As the next digit (5) is 5 or more, the digit to the left of 5 needs to be increased by one. As this is a 9, the next higher number is 10, so the previous digit also needs to change to the next higher number. Write your answer.  $= 94.\overline{738}30$  (to five decimal places)

## Exercise 1D

**Example 4** 1 Round off to the nearest whole number.

- a** 87.15                      **b** 605.99                      **c** 2.5                      **d** 33.63

**Example 4** 2 Round off to the nearest hundred.

- a** 6827                      **b** 46 770                      **c** 79 999                      **d** 313.4

**Example 6** 3 Write these scientific notation numbers as basic numerals.

- a**  $5.3467 \times 10^4$                       **b**  $3.8 \times 10^6$                       **c**  $7.89 \times 10^5$                       **d**  $9.21 \times 10^{-3}$   
**e**  $1.03 \times 10^{-7}$                       **f**  $2.907 \times 10^6$                       **g**  $3.8 \times 10^{-12}$                       **h**  $2.1 \times 10^{10}$

**Example 5** 4 Write these numbers in scientific notation.

- a** 792 000                      **b** 14 600 000                      **c** 500 000 000 000                      **d** 0.000 009 8  
**e** 0.145 697                      **f** 0.000 000 000 06                      **g** 2 679 886                      **h** 0.0087

5 Express the following approximate numbers, using scientific notation.

- a** The mass of the Earth is 6 000 000 000 000 000 000 000 kg.  
**b** The circumference of the Earth is 40 000 000 m.  
**c** The diameter of an atom is 0.000 000 000 1 m.  
**d** The radius of the Earth's orbit around the Sun is 150 000 000 km.



Example 7, 8

6 For each of the following numbers, state the number of significant figures.

- a** 89 156                      **b** 608 765                      **c** 900 000 000 000      **d** 0.709  
**e** 0.10                      **f** 0.006                      **g** 450 000                      **h** 0.008 007

7 Write the following correct to the number of significant figures indicated in each of the brackets.

- a** 4.8976                      (2)                      **b** 0.078 74                      (3)  
**c** 1506.892                      (5)                      **d** 5.523                      (1)

8 Calculate the following and give your answer correct to the number of significant figures indicated in each of the brackets.

- a**  $4.3968 \times 0.000\ 743\ 8$                       (2)      **b**  $0.611\ 35 \div 4.1119$                       (5)  
**c**  $3.4572 \div 0.0109$                       (3)      **d**  $50\ 042 \times 0.0067$                       (3)

Example 9

9 Use a calculator to find answers to the following. Give each answer correct to the number of decimal places indicated in the brackets.

- a**  $3.185 \times 0.49$                       (2)      **b**  $0.064 \div 2.536$                       (3)  
**c**  $0.474 \times 0.0693$                       (2)      **d**  $12.943 \div 6.876$                       (4)  
**e**  $0.006\ 749 \div 0.000\ 382$                       (3)      **f**  $38.374\ 306 \times 0.007\ 493$                       (4)



10 Calculate the following, correct to two decimal places.

- a**  $\sqrt{7^2 + 14^2}$                       **b**  $\sqrt{3.9^2 + 2.6^2}$                       **c**  $\sqrt{48.71^2 - 29^2}$

## 1E Conversion of units

The modern metric system in Australia is defined by the International System of Units (SI), which is a system of measuring and has three main units.

### The three main SI units of measurement

- m      the *metre* for length  
kg      the *kilogram* for mass  
s      the *second* for time

Larger and smaller units are based on these by the addition of a prefix. When solving problems, we need to ensure that the units we use are the same. We may also need to convert our answer into specified units.

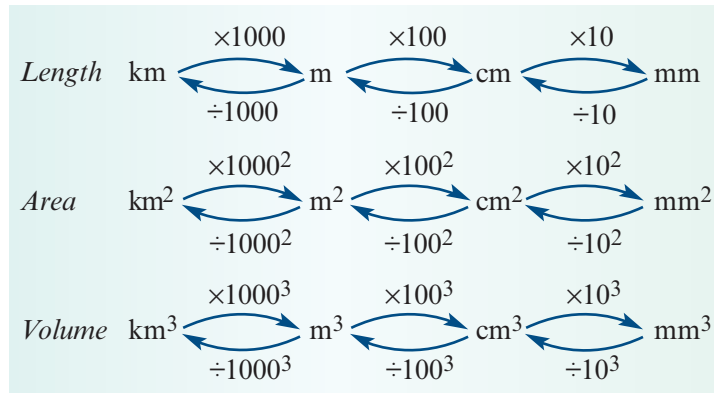


**Conversion of units**

To convert units remember to:

- use multiplication ( $\times$ ) when you convert from a larger unit to a smaller unit
- use division ( $\div$ ) when you convert from a smaller unit to a larger unit.

The common units used for measuring *length* are kilometres (km), metres (m), centimetres (cm) and millimetres (mm). The following chart is useful when converting units of length, and can be adapted to other metric units.



The common units for measuring *liquids* are kilolitres (kL), litres (L) and millilitres (mL).

$$1 \text{ kilolitre} = 1000 \text{ litres}$$

$$1 \text{ litre} = 1000 \text{ millilitres}$$

The common units for measuring *mass* are tonnes (t), kilograms (kg), grams (g) and milligrams (mg).

$$1 \text{ tonne} = 1000 \text{ kilograms}$$

$$1 \text{ kilogram} = 1000 \text{ grams}$$

$$1 \text{ gram} = 1000 \text{ milligrams}$$

**Note:** Strictly speaking the litre and tonne are not included in the SI, but are commonly used with SI units.

The following prefixes are useful to remember.

Prefix	Symbol	Definition	Decimal
micro	$\mu$	millionth	0.000 001
milli	m	thousandth	0.001
centi	c	hundredth	0.01
deci	d	tenth	0.1
kilo	k	thousand	1000
mega	M	million	1 000 000
giga	G	billion	1 000 000 000

**Example 10** Converting between units

Convert these measurements into the units given in the brackets.

**a** 5.2 km (m)

**b** 339 cm<sup>2</sup> (m<sup>2</sup>)

**c** 9.75 cm<sup>3</sup> (mm<sup>3</sup>)

**Solution**

**a** As there are 1000 metres in a kilometre and we are converting from kilometres (km) to a smaller unit (m), we need to multiply 5.2 by 1000.

$$5.2 \times 1000 \\ = 5200 \text{ m}$$

**b** As there are 100<sup>2</sup> square centimetres in a square metre and we are converting from square centimetres (cm<sup>2</sup>) to a larger unit (m<sup>2</sup>), we need to divide 339 by 100<sup>2</sup>.

$$339 \div 100^2 \\ = 0.039 \text{ m}^2$$

**c** As there are 10<sup>3</sup> cubic millimetres in a cubic centimetre and we are converting from cubic centimetres (cm<sup>3</sup>) to a smaller unit (mm<sup>3</sup>), we need to multiply 9.75 by 10<sup>3</sup>.

$$9.75 \times 10^3 \\ = 9750 \text{ mm}^3$$

Sometimes a measurement conversion requires more than one step.

**Example 11** Converting between units requiring more than one step

Convert these measurements into the units given in the brackets.

**a** 40 000 cm (km)

**b** 0.000 22 km<sup>2</sup> (cm<sup>2</sup>)

**c** 0.08m<sup>3</sup> (mm<sup>3</sup>)

**Solution**

**a** As there are 100 centimetres in a metre and 1000 metres in a kilometre and we are converting from centimetres (cm) to a larger unit (km), we need to divide 40 000 by (100 × 1000) = 100 000.

$$40\,000 \div 100\,000 \\ = 0.4 \text{ km}$$

**b** As there are 100<sup>2</sup> square centimetres in a square metre and 1000<sup>2</sup> square metres in a square kilometre and we are converting from square kilometres (km<sup>2</sup>) to a smaller unit (cm<sup>2</sup>), we need to multiply 0.000 22 by (100<sup>2</sup> × 1000<sup>2</sup>).

$$0.000\,22 \times 100^2 \times 1000^2 \\ = 2\,200\,000 \text{ cm}^2$$

**c** As there are 10<sup>3</sup> cubic millimetres in a cubic centimetre and 100<sup>3</sup> cubic centimetres in a cubic metre and we are converting from cubic metres (m<sup>3</sup>) to a smaller unit (mm<sup>3</sup>), we need to multiply 0.08 by (10<sup>3</sup> × 100<sup>3</sup>).

$$0.08 \times 10^3 \times 100^3 \\ = 80\,000\,000 \text{ mm}^3$$

## Exercise 1E

Example 10

1 Convert the following measurements into the units given in brackets.

- a** 5.7 m (cm)      **b** 1.587 km (m)      **c** 8 cm (mm)      **d** 670 cm (m)  
**e** 0.0046 km (cm)      **f** 289 mm<sup>2</sup> (cm<sup>2</sup>)      **g** 5.2 m<sup>2</sup> (cm<sup>2</sup>)      **h** 0.08 km<sup>2</sup> (m<sup>2</sup>)  
**i** 3700 mm<sup>2</sup> (cm<sup>2</sup>)      **j** 6 m<sup>2</sup> (mm<sup>2</sup>)      **k** 500 mL (L)      **l** 0.7 kg (g)  
**m** 2.3 kg (mg)      **n** 567 000 mL (kL)      **o** 793 400 mg (g)      **p** 75.5 kg (mg)  
**q** 0.5 L (mL)

Example 11

2 Convert the following measurements into the units indicated in brackets and give your answer in standard form.

- a** 5 tonne (kg)      **b** 6000 mg (kg)      **c** 27 100 km<sup>2</sup> (m<sup>2</sup>)      **d** 33 m<sup>3</sup> (cm<sup>3</sup>)  
**e** 487 m<sup>2</sup> (km<sup>2</sup>)      **f** 28 mL (L)      **g** 6 km (cm)      **h** 1125 mL (kL)  
**i** 50 000 m<sup>3</sup> (km<sup>3</sup>)      **j** 340 000 mm<sup>3</sup> (m<sup>3</sup>)

3 Find the total sum of these measurements. Express your answer in the units given in brackets.

- a** 14 cm, 18 mm (mm)      **b** 589 km, 169 m (km)  
**c** 3.4 m, 17 cm, 76 mm (cm)      **d** 300 mm<sup>2</sup>, 10.5 cm<sup>2</sup> (cm<sup>2</sup>)

4 A wall in a house is 7860 mm long. How many metres is this?

5 A truck weighs 3 tonne. How heavy is this in kilograms?

6 An Olympic swimming pool holds approximately 2.25 megalitres of water. How many litres is this?



7 Baking paper is sold on a roll 30 cm wide and 10 m long. How many baking trays of width 30 cm and length 32 cm could be covered with one roll of baking paper?

## 1F Percentages

Per cent is an abbreviation of the Latin words *per centum*, which mean ‘by the hundred’.

A **percentage** is a rate or a proportion expressed as part of one hundred. The symbol used to indicate percentage is  $\%$ . Percentages can be expressed as common fractions or as decimals.

For example: 17% (17 per cent) means 17 parts out of every 100.

$$17\% = \frac{17}{100} = 0.17$$



**Conversions**

- 1 To convert a fraction or a decimal to a percentage, multiply by 100.
- 2 To convert a percentage to a decimal or a fraction, divide by 100.

**Example 12** Converting fractions to percentages

Express  $\frac{36}{90}$  as a percentage.

**Solution**

Method 1 (by hand)

- 1 Multiply the fraction  $\frac{36}{90}$  by 100.
- 2 Evaluate and write your answer.

$$\begin{aligned}\frac{36}{90} \times 100 \\ = 40\%\end{aligned}$$

*Note:* The above calculation can be performed on the ClassPad calculator.

Method 2 (using CAS)

- 1 Enter  $36 \div 90$  on calculator.
- 2 Press % sign and EXE (Casio) or ENTER (Ti-Nspire).
- 3 Write your answer.

36/90%	40
--------	----

Expressed as a percentage,  
 $\frac{36}{90}$  is 40%.

**Example 13** Converting a decimal to a percentage

Express 0.75 as a percentage.

**Solution**

- 1 Multiply 0.75 by 100.
- 2 Evaluate and write your answer.

$$\begin{aligned}0.75 \times 100 \\ = 75\%\end{aligned}$$

**Example 14** Converting a percentage to a fraction

Express 62% as a common fraction.

**Solution**

- 1 As 62% means 62 out of 100, this can be written as a fraction  $\frac{62}{100}$ .
- 2 Simplify the fraction by dividing both the numerator and the denominator by 2.

$$\begin{aligned}62\% &= \frac{62}{100} \\ &= \frac{62 \div 2}{100 \div 2} \\ &= \frac{31}{50}\end{aligned}$$

**Example 15** Converting a percentage to a decimal

Express 72% as a decimal.

**Solution**

- 1 Write 72% as a fraction over 100 and express this as a decimal.

$$\frac{72}{100} = 0.72$$

► **Finding a percentage of a quantity**

To find a percentage *of* a number or a quantity, remember that in mathematics *of* means *multiply*.

**Example 16** Finding a percentage of a quantity

Find 15% of \$140.

**Solution****Method 1**

- 1 Write out problem and rewrite 15% as a fraction out of 100.

$$15\% \text{ of } 140$$

$$= \frac{15}{100} \text{ of } 140$$

- 2 Change *of* to *multiply*.

$$= \frac{15}{100} \times 140$$

- 3 Perform the calculation and write your answer.

$$= 21$$

**Note:** The above calculation can be performed on the CAS calculator.

**Method 2 (using CAS)**

- 1 Enter 15%140 on calculator.  
2 Press EXE (Casio) or ENTER (Ti-Nspire).  
3 Write your answer.

15%140

21

21

► **Comparing two quantities**

One quantity or number may be expressed as a percentage of another quantity or number (both quantities must always be in the same units). Divide the quantity by what you are comparing it with and then multiply by 100 to convert it to a percentage.



**Example 17** Expressing a quantity as a percentage of another quantity

There are 18 girls in a class of 25 students. What percentage of the class are girls?

**Solution**

- |  |  |
|--|--|
| <b>1</b> Work out the fraction of girls in the class.                | $\text{Girls} = \frac{18}{25}$               |
| <b>2</b> Convert the fraction to a percentage by multiplying by 100. | $\frac{18}{25} \times 100$                   |
| <b>3</b> Evaluate and write your answer.                             | $= 72$<br><i>72% of the class are girls.</i> |

**Example 18** Expressing a quantity as a percentage of another quantity with different units

Express 76 mm as a percentage of 40 cm.

**Solution**

- |   |  |
|---|--|
| <b>1</b> First convert 40 centimetres to millimetres by multiplying by 10, as there are 10 millimetres in 1 centimetre. | $40 \text{ cm} = 40 \times 10$<br>$= 400 \text{ mm}$ |
| <b>2</b> Write 76 millimetres as a fraction of 400 millimetres.   | $\frac{76}{400}$                                     |
| <b>3</b> Multiply by 100 to convert to a percentage.  | $\frac{76}{400} \times 100$                          |
| <b>4</b> Evaluate and write your answer.  | $= 19\%$   |

**Exercise 1F****Example 12, 13**

**1** Express the following as percentages.

**a**  $\frac{1}{4}$

**b**  $\frac{2}{5}$

**c**  $\frac{3}{20}$

**d**  $\frac{7}{10}$

**e** 0.19

**f** 0.79

**g** 2.15

**h** 39.57

**i** 0.073

**j** 1

**Example 14, 15**

**2** Express the following as:

**i** common fractions, in their lowest terms**ii** decimals.

**a** 25%

**b** 50%

**c** 75%

**d** 68%

**e** 5.75%

**f** 27.2%

**g** 0.45%

**h** 0.03%

**i** 0.0065%

**j** 100%

**Example 16** 3 Find the following, correct to three significant figures.

- |                             |  |
|-----------------------------|--|
| <b>a</b> 15% of \$760       | <b>b</b> 22% of \$500                  |
| <b>c</b> 17% of 150 m       | <b>d</b> $13\frac{1}{2}\%$ of \$10 000 |
| <b>e</b> 2% of 79.34 cm     | <b>f</b> 19.6% of 13.46                |
| <b>g</b> 0.46% of 35 €      | <b>h</b> 15.9% of \$28 740             |
| <b>i</b> 22.4% of \$346 900 | <b>j</b> 1.98% of \$1 000 000          |

**Example 17** 4 From a class, 28 out of 35 students wanted to take part in a project. What percentage of the class wanted to take part?

- 5 A farmer lost 450 sheep out of a flock of 1200 during a drought. What percentage of the flock were lost?



- 6 In a laboratory test on 360 light globes, 16 globes were found to be defective. What percentage were satisfactory, correct to one decimal place?
- 7 After three rounds of a competition, a basketball team had scored 300 points and 360 points had been scored against them. Express the points scored by the team as a percentage of the points scored against them. Give your answer correct to two decimal places.
- 8 In a school of 624 students, 125 are in year 10. What percentage of the students are in year 10? Give your answer to the nearest whole number.

**Example 18** 9 Express 75 cm as a percentage of 2 m.

- 10 In a population of  $3\frac{1}{4}$  million people, 2 115 000 are under the age of 16. Calculate the percentage, to two decimal places, of the population who are under the age of 16.
- 11 The cost of producing a chocolate bar that sells for \$1.50 is 60c. Calculate the profit made on a bar of chocolate as a percentage of the production cost of a bar of chocolate.



## 1G Percentage increase and decrease

When increasing or decreasing a quantity by a given percentage, the percentage increase or decrease is always calculated as a percentage of the *original* quantity.



### Example 19 Calculating the new price following a percentage increase

Sally's daily wage of \$175 is increased by 15%. Calculate her new daily wage.

#### Solution

##### Method 1

- |  |  |
|--|--|
| <b>1</b> First find 15% of \$175 by rewriting 15% as a fraction out of 100 and changing <i>of</i> to multiply (or use a calculator). | $15\% \text{ of } 175$ $= \frac{15}{100} \times 175$ $= 26.25$ |
| <b>2</b> Perform the calculation and write your answer.  |  |
| <b>3</b> As \$175 is to be increased by 15%, add \$26.25 to the original amount of \$175.  | $175 + 26.25$ $= 201.25$                                       |
| <b>4</b> Write your answer in a sentence.  | <p><i>Sally's new daily wage is \$201.25.</i></p>              |

##### Method 2

- |  |   |
|--|---|
| <b>1</b> An increase of 15% means that the new amount will be the original amount (in other words, 100%) plus an extra 15%.<br>Find 115% of 175. | $115\% \text{ of } 175$ $= \frac{115}{100} \times 175$ $= 201.25$ |
| <b>2</b> Perform the calculation.  |   |
| <b>3</b> Write your answer in a sentence.  | <p><i>Sally's new daily wage is \$201.25.</i></p>                 |

**Example 20** Calculating the new amount following a percentage decrease

A primary school's fun run distance of 2.75 km is decreased by 20% for students in years 2 to 4. Find the new distance.

**Solution****Method 1**

- |   |  |
|---|--|
| <b>1</b> First find 20% of 2.75 by writing 20% as a fraction out of 100 and changing <i>of</i> to multiply (or use a calculator). | $\begin{aligned} &20\% \text{ of } 2.75 \\ &= \frac{20}{100} \times 2.75 \\ &= 0.55 \end{aligned}$ |
| <b>2</b> Evaluate and write your answer.  | $= 0.55$   |
| <b>3</b> As 2.75 km is to be decreased by 20%, subtract 0.55 km from the original 2.75 km.  | $\begin{aligned} &2.75 - 0.55 \\ &= 2.2 \end{aligned}$   |
| <b>4</b> Write your answer in a sentence.   | <i>The new distance is 2.2 km.</i>   |

**Method 2**

- |  |   |
|--|---|
| <b>1</b> A decrease of 20% means that the new amount will be the original amount (100%) minus 20%. Find 80% of 2.75. | $\begin{aligned} &80\% \text{ of } 2.75 \\ &= \frac{80}{100} \times 2.75 \\ &= 2.2 \end{aligned}$ |
| <b>2</b> Perform the calculation.  |   |
| <b>3</b> Write your answer in a sentence.  | <i>The new distance is 2.2 km.</i>  |

**Example 21** Calculating a new price with a percentage discount

If a shop offers a discount of 15% on items in a sale, what would be the sale price of a pair of jeans originally priced at \$95?

**Solution****Method 1**

- |  |   |
|--|---|
| <b>1</b> Find 15% of 95.   | $\begin{aligned} 15\% \text{ of } 95 &= \frac{15}{100} \times 95 \\ &= 14.25 \end{aligned}$ |
| <b>2</b> As jeans are discounted by 15%, this is a decrease, so we need to subtract the discounted price of \$14.25 from the original price of \$95. | $\begin{aligned} &95 - 14.25 \\ &= 80.75 \end{aligned}$                                     |
| <b>3</b> Write your answer in a sentence.  | <i>The sale price would be \$80.75</i>  |

**Method 2**

- |  |   |
|--|---|
| <b>1</b> A discount of 15% means that the new amount is 85% of 95. | $\begin{aligned} &85\% \text{ of } 95 \\ &= \frac{85}{100} \times 95 \\ &= 80.75 \end{aligned}$ |
| <b>2</b> Perform the calculation.                                  | $= 80.75$   |
| <b>3</b> Write your answer in a sentence.                          | <i>The sale price would be \$80.75</i>  |

## ► Finding a percentage change

If we are given the original price and the new price of an item, we can find the percentage change. To find percentage change, we compare the change (increase or decrease) with the original number.

### Percentage change

$$\text{Percentage change} = \frac{\text{change}}{\text{original}} \times 100$$

Thus:

$$\text{Percentage discount} = \frac{\text{discount}}{\text{original}} \times 100$$

$$\text{Percentage increase} = \frac{\text{increase}}{\text{original}} \times 100$$



### Example 22 Calculating a percentage increase

A university increased its total size at the beginning of an academic year by 3000 students. If the previous number of students was 35 000, by what percentage, correct to two decimal places, did the student population increase?

#### Solution

- 1** To find the percentage increase, use the formula:

$$\text{Percentage increase} = \frac{\text{increase}}{\text{original}} \times 100$$

Substitute increase as 3000 and original as 35 000.

- 2** Evaluate.  
**3** Write your answer correct to two decimal places.

$$\text{Percentage increase} = \frac{\text{increase}}{\text{original}} \times 100$$

$$= \frac{3000}{35\,000} \times 100$$

$$= 8.5714 \dots$$

Student population increased by 8.57%.





### Example 23 Calculating the percentage discount

Calculate the percentage discount obtained when a calculator with a normal price of \$38 is sold for \$32 to the nearest whole per cent.

#### Solution

- 1** Find the amount of discount given by subtracting the new price, \$32, from the original price \$38.

$$\begin{aligned} \text{Discount} &= \$38 - \$32 \\ &= \$6 \end{aligned}$$

- 2** To find the percentage discount, use formula:

$$\text{Percentage discount} = \frac{\text{discount}}{\text{original}} \times 100$$

Substitute discount as 6 and original as 38 and evaluate.

$$\text{Percentage discount} = \frac{\text{discount}}{\text{original}} \times 100$$

$$= \frac{6}{38} \times 100$$

$$= 15.7895 \dots$$

- 3** Write your answer to the nearest whole per cent.

The percentage discount is 16%.

## Exercise 1G

### Example 21

- 1** A jewellery store has a promotion of 20% discount on all watches.

- a** How much discount will you get on a watch marked \$185?  
**b** What is the sale price of the watch?



- 2** A store gave different savings discounts on a range of items in a sale. Copy and complete the following table.

	Normal price	% Discount	Saving	Sale price
<b>a</b>	\$89.99	5		
<b>b</b>	\$189.00	10		
<b>c</b>	\$499.00	15		
<b>d</b>	\$249.00	20		
<b>e</b>	\$79.95	22.5		
<b>f</b>	\$22.95	25		
<b>g</b>	\$599.00	27.5		
<b>h</b>	\$63.50	30		
<b>i</b>	\$1000.00	33		

- 3** In a particular shop the employees are given a  $12\frac{1}{2}\%$  discount on any items they purchase. Calculate the actual price an employee would pay for each of the following:
- a** \$486 laptop  
**b** \$799 HD LED television  
**c** \$260 iPod  
**d** \$750 digital camera  
**e** \$246 digital video recorder

- 4** A clothing store offers 6% discount for cash sales. A customer who paid cash purchased the following items:  
 One pair of jeans \$95.95  
 A leather belt at \$29.95  
 Two jumpers at \$45 each  
 Calculate:
- a** the total saving  
**b** the actual amount paid for the goods.



- 5** Which results in the larger sum of money, increasing \$50 by 10% or decreasing \$60 by 8%?

**Example 19**

- 6** The production of a particular model of car is increased from 14 000 by 6% over a 12-month period. What is the new production figure?
- 7** If a new car is sold for \$23 960 and three years later it is valued at \$18 700, calculate the percentage depreciation, correct to two decimal places.

**Example 22**

- 8** A leading tyre manufacturer claims that a new tyre will average 12% more life than a previous tyre. The owner of a taxi fleet finds that the previous tyre averaged 24 000 km before replacement. How many kilometres should the new tyre average?

**Example 23**

- 9** Calculate the percentage discount for each of the following, to the nearest whole number.

	Normal price	Selling price	% Discount
<b>a</b>	\$60.00	\$52.00	
<b>b</b>	\$250.00	\$185.00	
<b>c</b>	\$5000.00	\$4700.00	
<b>d</b>	\$3.80	\$2.90	
<b>e</b>	\$29.75	\$24.50	
<b>f</b>	\$12.95	\$10.00	

- 10** A second-hand car advertised for sale at \$13 990 was sold for \$13 000. Calculate, correct to two decimal places, the percentage discount obtained by the purchaser.

- 11** A sport shop advertised the following items in their end-of-year sale. Calculate the percentage discount for each of the items to the nearest whole number.

	Normal price	Selling price	% Discount
<b>a</b>	Shoes	\$79.99	\$65.00
<b>b</b>	12 pack of golf balls	\$29.99	\$19.99
<b>c</b>	Exercise bike	\$1099.00	\$599.00
<b>d</b>	Basket ball	\$49.99	\$39.99
<b>e</b>	Sports socks	\$14.95	\$10.00
<b>f</b>	Hockey stick	\$299.00	\$250.00

- 12** Find the percentage increase that has been applied in each of the following:

- a** a book that is increased from \$20 to \$25  
**b** an airfare that is increased from \$300 to \$420  
**c** accommodation costs that are increased from \$540 to \$580.50.



## 1H Ratio and proportion

Ratios are used to numerically compare the values of two or more quantities.

A *ratio* can be written as **a : b** (read as 'a to b'). It can also be written as a fraction  $\frac{a}{b}$ .

The order of the numbers or numerals in a ratio is important.  $a : b$  is *not* the same as  $b : a$

### Example 24 Expressing quantities as a ratio

In a year 10 class of 26 students there are 14 girls and 12 boys. Express the number of girls to boys as a ratio.

#### Solution

As there are 14 girls and 12 boys, the ratio of girls to boys is 14 : 12.

Note: This could also be written as a fraction  $\frac{14}{12}$ .

### Example 25 Expressing more than two quantities as a ratio

A survey of the same group of 26 students showed that 10 students walked to school, 11 came by public transport, and 5 were driven by their parents. Express as a ratio the number of students who walked to school to the number of students who came by public transport to the number of students who were driven to school.

#### Solution

The order of the numbers in a ratio is important.

10 students walked, 11 used public transport and 5 were driven so the ratio is 10 : 11 : 5.

**Example 26** Expressing quantities as a ratio

A cordial bottle has instructions to mix  
1 part cordial with 4 parts water.  
Express this as a ratio.

**Solution**

The ratio of cordial to water is 1 : 4. This could also be written as  $\frac{1}{4}$ .

Note: The reverse ratio of water to cordial is 4 : 1, which could also be written as  $\frac{4}{1}$ .

**Exercise 1H****Example 24**

- 1 A survey of a group of 50 year 11 students in a school showed that 35 of them have a part-time job and 15 do not. Express the number of students having a part-time job to those who do not as a ratio.

**Example 25**

- 2 The table below shows the average life expectancy of some animals.

Animal	Life expectancy
Chimpanzee	40 years
Elephant	70 years
Horse	40 years
Kangaroo	9 years
Tortoise	120 years
Mouse	4 years
Whale	80 years



Find the ratios between the life expectancies of the following animals.

- Whale to horse
- Elephant to kangaroo
- Whale to tortoise
- Chimpanzee to mouse
- Horse to mouse to whale



## 11 Expressing ratios in their simplest form

Ratios can be simplified by dividing through by a common factor or by multiplying each term as required.

### Example 27 Simplifying ratios

Simplify the follow ratios.

**a**  $15 : 20$

**b**  $0.4 : 1.7$

**c**  $\frac{3}{4} : \frac{5}{3}$

#### Solution

**a 1** Divide both 15 and 20 by 5.

$$15 : 20$$

**2** Evaluate and write your answer.

$$= \frac{15}{5} : \frac{20}{5}$$

$$= 3 : 4$$

**b 1** Multiply both 0.4 and 1.7 by 10 to give whole numbers.

$$0.4 : 1.7$$

**2** Evaluate and write your answer.

$$= 0.4 \times 10 : 1.7 \times 10$$

$$= 4 : 17$$

**c Method 1**

**1** Multiply both fractions by 4.

$$\frac{3}{4} \times 4 : \frac{5}{3} \times 4$$

$$= 3 : \frac{20}{3}$$

**2** Multiply both sides of the equation by 3.

$$= 3 \times 3 : \frac{20}{3} \times 3$$

**3** Write your answer.

$$= 9 : 20$$

**Method 2**

**1** Multiply both  $\frac{3}{4}$  and  $\frac{5}{3}$  by the lowest common multiple (LCM) of 3 and 4, which is 12, to eliminate fractions.

$$\frac{3}{4} : \frac{5}{3}$$

$$= \frac{3}{4} \times 12 : \frac{5}{3} \times 12$$

**2** Evaluate and write your answer.

$$= 9 : 20$$

In each of the above examples, the ratios are equivalent and the information is unchanged. For example, the ratio:

$12 : 8$  is equivalent to the ratio  $24 : 16$  (multiply both 12 and 8 by 2)

and

$12 : 8$  is also equivalent to the ratio  $3 : 2$  (divide both 12 and 8 by 4)

**Ratios**

- 1 When ratios are written in terms of the smallest possible whole numbers, they are expressed in their *simplest form*.
- 2 The order of the figures in a ratio is important.  $3 : 5$  is *not* the same as  $5 : 3$ .
- 3 Both parts of a ratio must be expressed in the same unit of measurement.

**Example 28** Simplifying ratios with different units

Express 15 cm to 3 m as a ratio in its simplest form.

**Solution**

- 1 Write down the ratio.  $15 \text{ cm} : 3 \text{ m}$
- 2 Convert 3 m to cm, by multiplying 3 m by 100, so that both parts of the ratio will be in the same units.  $15 \text{ cm} : 3 \times 100 \text{ cm}$   
 $= 15 \text{ cm} : 300 \text{ cm}$
- 3 Simplify the ratio by dividing both 15 and 300 by 15.  $= 15 : 300$   
 $= \frac{15}{15} : \frac{300}{15}$
- 4 Write your answer.  $= 1 : 20$

**Example 29** Finding missing values in a ratio

Find the missing value for the equivalent ratios  $3 : 7 = \square : 28$ .

**Solution**

- 1 Let the unknown value be  $x$  and write the ratios as fractions.  $3 : 7 = x : 28$
- 2 Solve for  $x$ .  $\frac{3}{7} = \frac{x}{28}$

**Method 1 (by hand)**

- 1 Multiply both sides of equation by 28.  $\frac{3}{7} \times 28 = \frac{x}{28} \times 28$
- 2 Evaluate and write your answer.  $x = 12$   
 $3 : 7 = 12 : 28$

**Method 2 (using CAS)**

Use the solve function.

$$\text{solve}\left(\frac{3}{7} = \frac{x}{28}, x\right)$$

$$x = 12$$

## Exercise 11

**Example 27**

**1** Express the following ratios in their simplest forms.

- a** 12 : 15                      **b** 10 : 45                      **c** 22 : 55 : 33                      **d** 1.3 : 3.9  
**e** 2.7 : 0.9                      **f**  $\frac{5}{3} : \frac{1}{4}$                       **g** 18 : 8

**Example 28**

**2** Express the following ratios in their simplest form after making sure that each quantity is expressed in the same units.

- a** 60 L to 25 L                      **b** \$2.50 to \$50                      **c** 75 cm to 2 m  
**d** 5 kg to 600 g                      **e** 15 mm to 50 cm to 3 m                      **f** 1 km to 1 m to 1 cm  
**g** 5.6 g to 91 g                      **h** \$30 to \$6 to \$1.20 to 60c

**Example 29**

**3** For each of the following equivalent ratios find the missing value.

- a** 1 : 4 =  : 20                      **b** 15 : 8 = 135 :                       **c** 600 : 5 =  : 1  
**d** 2 : 5 = 2000 :                       **e** 3 : 7 =  : 56

**4** Which of the following statements are true and which are false? For those that are false, suggest a correct replacement statement, if possible.

- a** The ratio 4 : 3 is the same as 3 : 4.  
**b** The ratio 3 : 4 is equivalent to 20 : 15.  
**c** 9 : 45 is equivalent to 1 : 5.  
**d** The ratio 60 to 12 is equivalent to 15 to 3, which is the same as 4 to 1.  
**e** If the ratio of a father's age to his daughter's age is 7 : 1, then the girl is 7 years old when her father is 56.  
**f** If my weekly allowance is  $\frac{5}{8}$  of that of my friend, then the ratio of my monthly allowance to the allowance of my friend is 20 : 32.

**5** The following recipe is for Anzac biscuits.

**Anzac biscuits (makes 25)**

100 grams rolled oats	60 grams desiccated coconut
175 grams plain all-purpose flour, sifted	125 grams soft brown sugar
125 grams butter	3 tablespoons boiling water
2 tablespoons golden syrup	1 teaspoon bicarbonate of soda



- a** What is the unsimplified ratio of rolled oats : coconut : flour : brown sugar : butter?  
**b** Simplify the ratio from part **a**.  
**c** You want to adapt the recipe to make 75 biscuits. What quantity of each ingredient do you need?





## 1J Dividing quantities in given ratios

### Example 30 Dividing quantities in given ratios

Calculate the number of students in each class if 60 students are divided into classes in the following ratios.

**a** 1 : 3

**b** 5 : 1

**c** 1 : 2 : 7

#### Solution

**a 1** Add up the total number of parts.  
(Remember that a 1 : 3 ratio means that there is 1 part for every 3 parts).

The total number of parts is  $1 + 3 = 4$ .

**2** Divide the number of students (60) by the number of parts (4) to give the number of students in one group.

$$60 \div 4 = 15$$

One group of students will have  $1 \times 15 = 15$  students.

**3** Work out how many students in the other group by multiplying the number of parts (3) by the number of students in one group (15).

The other group will have  $3 \times 15 = 45$  students.

**4** Check this gives a total of 60 students and write your answer.

$$15 + 45 = 60$$

The two groups will have 15 and 45 students.

**b 1** Add up the total number of parts.  
(Remember that a 1 : 5 ratio means that there is 1 part for every 5 parts).

The total number of parts is  $1 + 5 = 6$ .

**2** Divide the number of students (60) by the number of parts (6) to give the number of students in one group.

$$60 \div 6 = 10$$

One group of students will have  $1 \times 10 = 10$  students.

**3** Work out how many students in the other group by multiplying the number of parts (5) by the number of students in one group (10).

The other group will have  $5 \times 10 = 50$  students.

**4** Check this gives a total of 60 students and write your answer.

$$10 + 50 = 60.$$

The two groups will have 10 and 50 students.

- c 1** To divide 60 students into classes in the ratio  $1 : 2 : 7$ , first add up the total number of parts.
- 2** Divide the number of students (60) by the number of parts (10) to give the number of students in one group.
- 3** Work out how many students in the other two groups by multiplying the number of parts (2) and (7) by the number of students in one group (6).
- 4** Check that this gives 60 students and write your answer.

The total number of parts is  $1 + 2 + 7 = 10$ .

$$60 \div 10 = 6$$

One group of students will have  $1 \times 6 = 6$  students.

The other groups will have  $2 \times 6 = 12$  students and  $7 \times 6 = 42$  students.

$$6 + 12 + 42 = 60$$

The three groups will have 6, 12 and 42 students.

## Exercise 1J

Example 30a,b

- 1** If a 40 m length of rope is cut in the following ratios, what will be the lengths of the individual pieces of rope?
- a**  $4 : 1$                       **b**  $1 : 7$                       **c**  $60 : 20$   
**d**  $8 : 8$



Example 30c

- 2** If a sum of \$500 were shared among a group of people in the following ratios, how much would each person receive?
- a**  $6 : 4$                       **b**  $1 : 4 : 5$                       **c**  $1 : 8 : 1$                       **d**  $8 : 9 : 8$   
**e**  $10 : 5 : 4 : 1$

- 3** A basket contains bananas, mangos and pineapples in the ratio  $10 : 1 : 4$ . If there are 20 pineapples in the basket, calculate:
- a** the number of bananas  
**b** the number of mangos  
**c** the total amount of fruit in the basket.



- 4** 7.5 litres of cordial is required for a children's party. If the ratio of cordial to water is  $1 : 4$ :
- a** how many litres of cordial is required?  
**b** how many litres of water is required?



- 5** The scale on a map is  $1 : 20\,000$  (in cm). If the measured distance on the map between two historical markers is 15 centimetres, what is the actual distance between the two markers in kilometres?



## 1K Unitary method

Ratios can be used to calculate unit prices, i.e. the price of one item. This method is known as the *unitary method* and can be used to solve a range of ratio problems.

### Example 31 Using the unitary method

If 24 golf balls cost \$86.40, how much do 7 golf balls cost?


#### Solution

- |   |  |
|---|--|
| <p><b>1</b> Find the cost of 1 golf ball by dividing \$86.40 (the total cost) by 24 (the number of golf balls).</p> | $\$86.40 \div 24 = \$3.60$ $\$3.60 \times 7 = \$25.20$ |
| <p><b>2</b> Multiply the cost of one golf ball (\$3.60) by 7. Write your answer.</p>                                | <p>7 golf balls cost \$25.20</p>                       |

## Exercise 1K

### Example 31

- 1** Use the unitary method to answer the following questions.
  - a** If 12 cakes cost \$14.40, how much do 13 cakes cost?
  - b** If a clock gains 20 seconds in 5 days, how much does the clock gain in three weeks?
  - c** If 17 textbooks cost \$501.50, how much would 30 textbooks cost?
  - d** If an athlete can run 4.5 kilometres in 18 minutes, how far could she run in 40 minutes at the same pace?
  
- 2** If one tin of red paint is mixed with four tins of yellow paint, it produces five tins of orange paint. How many tins of the red and yellow paint would be needed to make 35 tins of the same shade of orange paint?
 


  
- 3** If a train travels 165 kilometres in 1 hour 50 minutes at a constant speed, calculate how far it could travel in:
 

<b>a</b> 3 hours	<b>b</b> $2\frac{1}{2}$ hours	<b>c</b> 20 minutes
<b>d</b> 70 minutes	<b>e</b> 3 hours and 40 minutes	<b>f</b> $\frac{3}{4}$ hour
  
- 4** Ice creams are sold in two different sizes. A 35 g cone costs \$1.25 and a 73 g cone costs \$2.00. Which is the better buy?
  
- 5** A shop sells 2 L containers of Brand A milk for \$2.99, 1 L of Brand B milk for \$1.95 and 600 mL of Brand C milk for \$1.42. Calculate the best buy.

- 6 You need 6 large eggs to bake 2 chocolate cakes. How many eggs will you need to bake 17 chocolate cakes?
- 7 A car uses 45 litres of petrol to travel 495 kilometres. Under the same driving conditions calculate:
- how far the car could travel on 50 litres of petrol
  - how much petrol the car would use to travel 187 kilometres.



## 1L Logarithms

Consider the numbers:

0.01, 0.1, 1, 10, 100, 1000, 10 000, 100 000, 1 000 000

Such numbers can be written more compactly as:

$10^{-2}$ ,  $10^{-1}$ ,  $10^0$ ,  $10^1$ ,  $10^2$ ,  $10^3$ ,  $10^4$ ,  $10^5$ ,  $10^6$

In fact, if we make it clear we are only talking about powers of 10, we can merely write down the powers:

-2, -1, 0, 1, 2, 3, 4, 5, 6

These powers are called the **logarithms** of the numbers or *logs* for short. When we use logarithms to write numbers as powers of 10, we say we are working with logarithms to the base 10.

### Powers of 10

$$10^7 = 10\,000\,000$$

$$10^6 = 1\,000\,000$$

$$10^5 = 100\,000$$

$$10^4 = 10\,000$$

$$10^3 = 1000$$

$$10^2 = 100$$

$$10^1 = 10$$

$$10^0 = 1$$

$$10^{-1} = 0.1$$

$$10^{-2} = 0.01$$

$$10^{-3} = 0.001$$

Knowing the powers of 10 is important when using logarithms to the base 10.

### Example 32 Evaluating a logarithm

Write the number 100 as a power of 10 and then write down its logarithm.

#### Solution

- 1 Write 100 as a power of 10.

$$100 = 10^2$$

- 2 Write down the logarithm.

$$\log(100) = \log(10^2)$$

$$= 2$$

### Example 33 Evaluating a logarithm giving a negative value

Write the number 0.001 as a power of 10 and then write down its logarithm.

#### Solution

- 1 Write 0.001 as a power of 10.

$$0.001 = 10^{-3}$$

- 2 Write down the logarithm.

$$\log(0.001) = \log(10^{-3})$$

$$= -3$$

**Example 34** Using a CAS calculator to find logs

Find the log of 45, correct to one decimal place.

**Solution**

- 1 Open a calculator screen, type  $\log(45)$  and press ENTER (Ti-Nspire) or EXE (Casio).
- 2 Write the answer correct to one decimal place.

$$\log_{10}(45) \quad 1.65321$$

$$\log(45) = 1.65 \dots$$

$$= 1.6 \text{ to one decimal place}$$

**Example 35** Using a CAS calculator to evaluate a number if log is known

Find the number whose log is 3.1876, correct to one decimal place.

**Solution**

- 1 If the log of a number is 3.1876, then the number is  $10^{3.1876}$ .
- 2 Enter the expression and press ENTER (Ti-Nspire) or EXE (Casio).
- 3 Write the answer correct to one decimal place.

$$10^{3.1876} \quad 1540.3$$

$$10^{3.1876} = 1540.281 \dots$$

$$= 1540.3 \text{ to one decimal place}$$

**Exercise 1L****Example 32, 33**

- 1 Write the number as a power of 10 and then write down its logarithm.
 

a 1000	b 1 000 000	c 0.0001	d 10 000 000
e 1	f 10	g 0.000 000 001	

**Example 34**

- 2 Use your calculator to evaluate, correct to three decimal places.
 

a $\log(300)$	b $\log(5946)$	c $\log(10\,390)$	d $\log(0.0047)$
e $\log(0.6)$	f $\log(0.089)$	g $\log(7.25)$	

**Determining numbers from logs****Example 35**

- 3 Find the numbers, correct to two decimal places, with logs of:
 

a 2.5	b $-1.5$
c 0.5	d 0



## 1M Order of magnitude

Increasing an object by an order of magnitude of 1 means that the object is ten times larger.

An increase of order of magnitude	Increase in size
1	$10^1 = 10$ times larger
2	$10^2 = 100$ times larger
3	$10^3 = 1000$ times larger
6	$10^6 = 1\,000\,000$ times larger

Decreasing an object by an order of magnitude 1 means that the object is ten times smaller.

An decrease of order of magnitude	Decrease in size
1	$10^{-1} = 0.1 = \frac{1}{10}$ smaller
2	$10^{-2} = 0.01 = \frac{1}{100}$ smaller
3	$10^{-3} = 0.001 = \frac{1}{1000}$ smaller
6	$10^{-6} = 0.000\,001 = \frac{1}{1\,000\,000}$ smaller

### An increase of order of magnitude

In general, an *increase* of  $n$  orders of magnitude is the equivalent of multiplying a quantity by  $10^n$ .

### A decrease of order of magnitude

In general, a *decrease* of  $n$  orders of magnitude is the equivalent of dividing a quantity by  $10^n$  or multiplying a quantity by  $10^{-n}$ .

It is easy to see the order of magnitude of various numbers when they are written in standard form (e.g. 200 in standard form is  $2 \times 100 = 2 \times 10^2$ ).

### Example 36 Finding the order of magnitude of a number written in standard form

What is the order of magnitude of 1200?

#### Solution

- Write 1200 in standard form.
- Look at the power of 10 to find the order of magnitude. Write your answer.

$$1200 = 1.2 \times 10^3$$

The power of 10 is 3 so the order of magnitude of 1200 is 3.

Note: The order of magnitude of 1.2 is 0.

## Exercise 1M

**Example 36**
**1** What is the order of magnitude of the following numbers?

**a** 46 000

**b** 559

**c** 3 000 000 000

**d**  $4.21 \times 10^{12}$ 
**e** 600 000 000 000

**2** A city has two TAFE colleges with 4000 students each. What is the order of magnitude of the total number of school students in the city?

**3** At the football stadium, 35 000 people attend a football match each week. What is the order of magnitude of the number of people who attend 8 weeks of games?

**4** A builder buys 9 boxes containing 1000 screws to build a deck.

**a** What is the order of magnitude of the total number of screws?

Once the deck is completed, the number of screws left is 90.

**b** What is the order of magnitude of the number of screws that are left?


## 1N Logarithmic scales

Some numbers in science are very large or very small.

		Scientific notation
Distance: Earth to Sun	150 000 000 km	$1.5 \times 10^8$ km
Distance: Earth to moon	384 000 km	$3.84 \times 10^5$ km
Mass: hydrogen atom	0.000 000 000 000 000 000 000 000 001 673 kg	$1.673 \times 10^{-27}$ kg
Wavelength: yellow light	0.000 000 55 m	$5.5 \times 10^{-7}$ m

Logarithmic scale graphs are useful when plotting a range of very small to very large numbers. Converting values to a logarithmic scale can make it easier to read and interpret values.



**Example 37** Converting values to logarithms in order to sketch a graph

Plot the heartbeat/minute of mammals against their body weight.

Animal	Body weight (g)	Heartbeat/minute
Shrew	2.5	0.40
Chick	50	400
Rabbit	1000	205
Monkey	5000	190
Tree kangaroo	8000	192
Giraffe	900 000	65
Elephant	5 000 000	30
Blue whale	170 000 000	16



**Note:** To plot the heartbeat/minute of mammals against their body weight, we will be starting from a very small weight value of 50 grams for a chick to 170 tonne = 170 000 kilograms = 170 000 000 grams for a blue whale.

Plotting the body weight values on a horizontal axis is difficult because of the large range of values for the body weight of mammals.

However, if the body weight values are written more compactly as logarithms (powers) of 10, then these logarithms can be placed on a logarithmic scale graph. For example, we have seen that  $\log_{10} 100 = 2$ . This can also be expressed as  $\log(100) = 2$ .

**Note:**  $\log_{10} x$  is often written as  $\log(x)$ .

**Solution**

- 1** Convert mammals' body weight to logarithms.

Weight of chick is 50 grams.

Find logarithm ( $\log$ ) of 50

50 is between 10 and 100

$$\log(10) = \log(10^1) = 1$$

$$\log(100) = \log(10^2) = 2$$

So  $\log(50)$  is between 1 and 2.

Use calculator to find  $\log(50)$ .

Weight of tree kangaroo is 8000 grams.

Find  $\log(8000)$ .

8000 is between 1000 and 10 000.

$$\log(1000) = \log(10^3) = 3$$

$$\log(50) = 1.70 \text{ (correct to two decimal places).}$$

$$\log(8000) = 3.90 \text{ (correct to two decimal places).}$$

$\log(10\ 000) = \log(10^4) = 4$

So  $\log(8000)$  must be between 3 and 4.

Use calculator to find  $\log(8000)$ .

Weight of giraffe is 900 000 grams.

Find  $\log(900\ 000)$ .

900 000 is between 100 000 and 1 000 000.

$\log(100\ 000) = \log(10^5) = 5$

$\log(1\ 000\ 000) = \log(10^6) = 6$

So  $\log(900\ 000)$  is between 5 and 6.

Use a calculator to  $\log(900\ 000)$

Weight of blue whale 170 000 000 grams (or  $1.7 \times 10^8$ ).

Use calculator to find  $\log(170\ 000\ 000)$

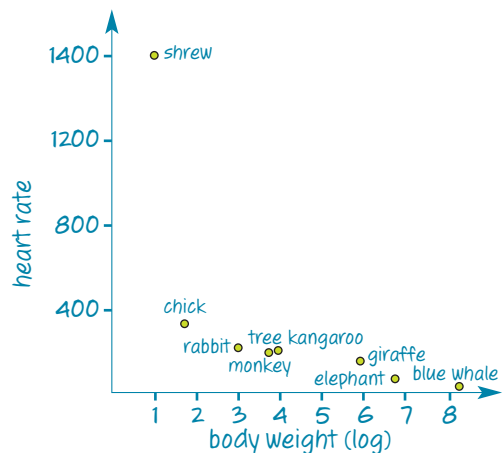
$\log(900\ 000) = 5.95$  (correct to two decimal places).

$\log(170\ 000\ 000) = 8.23$  (correct to two decimal places).

- 2 Use a calculator to find the logarithms (logs) of the body weight of the different mammals. Record your results.

Animal	Body weight (g)	Log (weight)
Shrew	2.5	0.40
Chick	50	1.70
Rabbit	1000	3.00
Monkey	5000	3.70
Tree kangaroo	8000	3.90
Giraffe	900 000	5.95
Elephant	5 000 000	6.70
Blue whale	170 000 000	8.23

- 3 Plot the logarithms of the animals' body weights on the horizontal axis of the graph and the heart rate on the vertical axis.



On a log scale:

- In moving from 1 to 2 we are actually increasing by a factor of 10.
- In moving from 2 to 3 we are increasing by a factor of 10.
- To go from 2 to 5 we will have multiplied by a factor of  $10 \times 10 \times 10 = 1000$ .

For example:

- The weight of the elephant is represented by the logarithm of 6.70 and the weight of the monkey is represented by the logarithm of 3.70. The difference between these logarithms is  $6.70 - 3.70 = 3$ , which means that the elephant is  $10^3 = 1000$  times heavier than the monkey.

Or, in a more complex situation:

- The weight of the rabbit is represented by the logarithm of 3 and the weight of the giraffe is represented by the logarithm of 5.95. The difference between these logarithms is  $5.95 - 3 = 2.95$  and represents  $10^{2.95} = 891.25$  indicating that the giraffe is 891.25 times heavier than a rabbit.

## ► Other real-life examples that use a logarithmic scale

### The earthquake magnitude scale

The strength of an earthquake is measured by the Moment Magnitude Scale (MMS), which takes the logarithm of the energy emitted by the quake. It is a modern modification of the earlier Richter Scale. The picture opposite shows the impact of the 2011 earthquake on the Christchurch Cathedral.



The numbers 1, 2, 3, 4, 5, 6, ... on the MMS indicate an intensity that is ten times stronger than the previous number. For example:

- A magnitude 5 earthquake is 10 times stronger than a magnitude 4 earthquake.
- A magnitude 6 earthquake is  $10 \times 10 = 100$  times stronger than a magnitude 4 earthquake.
- A magnitude 7 earthquake is  $10 \times 10 \times 10 = 1000$  times stronger than a magnitude 4 earthquake.

### Decibels (the loudness of sound)

When using the decibel scale to measure the loudness of sound, the least audible sound is assigned 0.

Thus:

- A sound 10 ( $= 10^1$ ) times louder than 0 is assigned a decibel value of 10.
- A sound 100 ( $= 10^2$ ) times louder than 0 is assigned a decibel value of 20.
- A sound 1000 ( $= 10^3$ ) times louder than 0 is assigned a decibel value of 30.
- A change in power by a factor of 10 corresponds to a 10 dB change in level.

**Example 38** Measuring the strength of an earthquake

The 2011 Tokyo earthquake was magnitude 9.0 on the MMS. To the nearest hundred, how much more intense was this compared to the 2011 Christchurch earthquake which was magnitude 6.3?

**Solution**

- |   |  |
|---|--|
| <b>1</b> Remembering that 9.0 and 6.3 are logarithmic values, subtract 6.3 from 9.0 | $9.0 - 6.3 = 2.7$  |
| <b>2</b> As this is a log value, evaluate $10^{2.7}$ .                              | $10^{2.7} = 501.187 \dots$   |
| <b>3</b> Round to the nearest hundred and write your answer.                        | <i>The Tokyo earthquake was 500 times stronger than the Christchurch earthquake.</i> |

**Example 39** Calculating the intensity of sound

The sound of a normal conversation is 60 decibels and the sound from sitting at the front row of a rock concert is 110 decibels. How much louder is the sound of the rock concert to the sound of normal conversation?

**Solution**

- |   |  |
|---|--|
| <b>1</b> First find out the difference in decibels by subtracting 60 from 110.  | $110 - 60 = 50$  |
| <b>2</b> Each increase of 10 decibels corresponds to 10 times the loudness. Divide 50 by $10 = 5$ , which corresponds to 5 lots of 10 times the loudness<br>$10 \times 10 \times 10 \times 10 \times 10 = 10^5$ . | $50 \div 10 = 5$<br><i>There are 5 lots of 10 decibels, which means</i><br>$10 \times 10 \times 10 \times 10 \times 10 = 10^5$ |
| <b>3</b> Evaluate $10^5$ .  | $10^5 = 100\,000$  |
| <b>4</b> Write your answer.   | <i>The sound at the front row of the rock concert is 100 000 times louder than normal conversation.</i>                        |

## Exercise 1N

- Example 38** 1 How many times stronger is a magnitude 7 earthquake than a magnitude 5 earthquake?
- 2 A magnitude 7.4 was recorded in the Solomon Islands in April 2014. Earlier that month, a magnitude 7.7 earthquake was recorded near the coast of Northern Chile. How much stronger than the Solomon Islands earthquake was the Chilean earthquake? Give your answer to the nearest whole number.
- 3 How much stronger is a magnitude 6.7 earthquake compared to one of 6.2? Give your answer correct to two decimal places.
- Example 37** 4 Use the logarithmic values for the animals' weights in Example 37 to find how much heavier than a shrew is a tree kangaroo, to the nearest thousand.
- Example 39** 5 If the sound of a normal conversation is 60 decibels, and the sound of a train going through a tunnel is 90 decibels, how much louder is the sound of the train than a conversation?



- 6 The sound of a vacuum cleaner is 80 decibels and someone whispering is 20 decibels. How much softer is the sound of someone whispering than the sound of a vacuum cleaner?

## Key ideas and chapter summary



### Order of operation

The order of operations is important. Remember BODMAS or BOMDAS

**B**rackets come first

**O**f or **O**rders (powers, square roots)

**D**ivision and **M**ultiplication come next, working from left to right then **A**ddition and **S**ubtraction, working from left to right

**Directed numbers** Multiplying or dividing two numbers with the **same** sign gives a **positive** value.

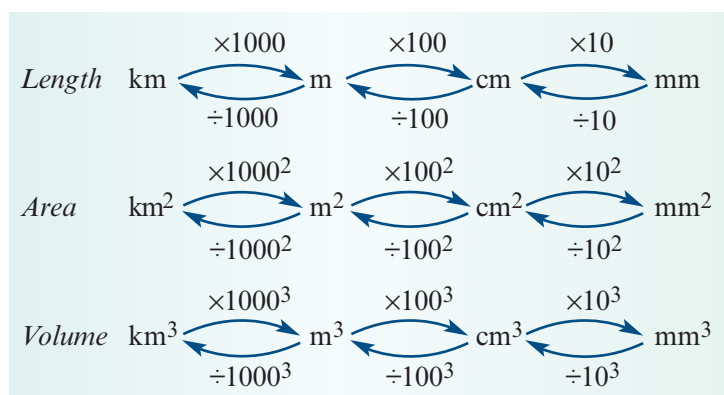
Multiplying or dividing two numbers with **different** signs gives a **negative** value.

**Scientific notation** To write a number in scientific notation express it as a number between 1 and 10 multiplied by a power of 10.

**Rounding** 5.417 rounded to two decimal places is 5.42 (number after the 1 is 7 so round up).

**Significant figures** All non-zero digits are significant.  
All zeroes between significant digits are significant.  
After a decimal point, all zeroes to the right of non-zero digits are significant.

### Conversion of measurements



1 kilolitre = 1000 litres

1 litre = 1000 millilitres

1 tonne = 1000 kilograms

1 kilogram = 1000 grams

1 gram = 1000 milligrams

### Percentages

To convert a fraction or a decimal to a percentage, multiply by 100.

To convert a percentage to a decimal or a fraction, divide by 100.

$$\text{Percentage change} = \frac{\text{change}}{\text{original quantity or price}} \times 100$$

<b>Ratios</b>	The order of the figures in a ratio is important. 4 : 3 is not the same as 3 : 4. Ratios can be simplified. Eg. $6 : 2 = 3 : 1$
<b>Logarithmic scales</b>	A logarithmic scale is often used to plot very large and/or very small numbers on a linear scale.

## Skills check

Having completed this chapter you should be able to:


- use a variety of mathematical operations in the correct order
- add, subtract, multiply and divide directed numbers
- find powers and roots of numbers
- round numbers to specific place values
- write numbers in scientific notation (standard form)
- understand and use significant figures
- convert units of measurements
- express ratios in their simplest form
- solve practical problems involving ratios, percentages and the unitary method
- use and interpret log scales when used to represent quantities that range over multiple orders of magnitude.

## Multiple-choice questions




- 1 Evaluate  $4 + 7 \times 3$ .  
**A** 33                      **B** 30                      **C** 19                      **D** 14                      **E** 25
- 2 Evaluate  $3 + (6 \div 3) - 2$ .  
**A** 3                      **B** 6                      **C** 1                      **D** 9                      **E** 8
- 3  $(8.7 - 4.9) \times (5.4 + 2.8)$  is equal to:  
**A** 23.32                      **B** 31.16                      **C** -14.96                      **D** 12.0                      **E** -31.48
- 4 Evaluate  $(-3) \times 4 \times 5$ .  
**A** 60                      **B** 6                      **C** -60                      **D** 27                      **E** 3
- 5 Evaluate  $(-2) + 8$ .  
**A** 10                      **B** 6                      **C** -10                      **D** -6                      **E** 28
- 6 Evaluate  $(-2) - (-3)$ .  
**A** -5                      **B** 5                      **C** 1                      **D** -1                      **E** 6



- 7** Evaluate  $5 - (-9)$   
**A**  $-4$       **B**  $59$       **C**  $44$       **D**  $-14$       **E**  $14$
- 8**  $3.895$  rounded to two decimal places is:  
**A**  $3.8$       **B**  $3.99$       **C**  $4.0$       **D**  $3.90$       **E**  $3.89$
- 9**  $4679$  rounded to the nearest hundred is:  
**A**  $5000$       **B**  $4600$       **C**  $4700$       **D**  $4670$       **E**  $4680$
- 10**  $5.21 \times 10^5$  is the same as:  
**A**  $52\,100\,000$       **B**  $521\,000$       **C**  $52\,105$       **D**  $0.000\,052\,1$       **E**  $260.50$
- 11**  $0.0048$  written in scientific notation is:  
**A**  $48 \times 10^{-4}$       **B**  $48 \times 10^{-3}$       **C**  $4.8 \times 10^3$       **D**  $4.8 \times 10^{-3}$       **E**  $4.8 \times 10^{-4}$
- 12**  $28\,037.2$  rounded to two significant figures is:  
**A**  $28\,000$       **B**  $20\,000.2$       **C**  $20\,007$       **D**  $7.2$       **E**  $28\,000.2$
- 13**  $0.030\,69$  rounded to two significant figures is:  
**A**  $0.03$       **B**  $0.000\,69$       **C**  $0.0307$       **D**  $0.031$       **E**  $0.0306$
- 14**  $5.1\text{ m}^2$  is the same as:  
**A**  $510\text{ cm}^2$       **B**  $0.0051\text{ km}^2$       **C**  $51\,000\text{ cm}^2$       **D**  $5100\text{ mm}^2$       **E**  $51\text{ cm}^2$
- 15**  $56\%$  as a fraction in its simplest form is:  
**A**  $0.56$       **B**  $\frac{56}{100}$       **C**  $\frac{0.56}{100}$       **D**  $\frac{5.6}{100}$       **E**  $\frac{28}{50}$
- 16**  $15\%$  of  $\$1600$  is equal to:  
**A**  $\$24$       **B**  $\$150$       **C**  $\$240$       **D**  $\$1840$       **E**  $\$24\,000$
- 17** An item with a cost price of  $\$450$  is marked up by  $30\%$ . Its selling price is:  
**A**  $\$585$       **B**  $\$135$       **C**  $\$480$       **D**  $\$1350$       **E**  $\$463.50$
- 18** A box contains 5 green marbles, 7 blue marbles and 3 yellow marbles. The ratio of blue marbles to total marbles is:  
**A**  $7 : 5 : 3$       **B**  $7 : 8$       **C**  $7 : 15$       **D**  $5 : 7 : 3$       **E**  $5 : 7 : 3 : 15$
- 19**  $\$750$  is divided in the ratio  $1 : 3 : 2$ . The smallest share is:  
**A**  $\$250$       **B**  $\$125$       **C**  $\$375$       **D**  $\$750$       **E**  $\$150$
-  **20** In simplest ratio form the ratio of 450 grams to 3 kilograms is:  
**A**  $3 : 20$       **B**  $450 : 3$       **C**  $9 : 60$       **D**  $150 : 1$       **E**  $15 : 100$

## Short-answer questions

- 1** Evaluate the following.
- a**  $3 + 2 \times 4$                       **b**  $25 \div (10 - 5) + 5$                       **c**  $14 - 21 \div 3$   
**d**  $(12 + 12) \div 12 + 12$                       **e**  $27 \div 3 \times 5 + 4$                       **f**  $4 \times (-2) + 3$   
**g**  $\frac{10 - 8}{2}$                       **h**  $\frac{4(3 + 12)}{2}$                       **i**  $\frac{-5 + 9}{2}$
- 2** Calculate the following and give your answer correct to two decimal places where appropriate.
- a**  $5^3$                       **b**  $\sqrt{64} - 5$                       **c**  $9^{\frac{1}{2}} + 9^{\frac{1}{2}}$                       **d**  $\sqrt{8}$   
**e**  $\sqrt{25 - 9}$                       **f**  $\sqrt{25} - 9$                       **g**  $\frac{6^3}{(10 \div 2)^2}$                       **h**  $\sqrt{6^2 + 10^2}$
- 3** Write each of the following in scientific notation.
- a** 2945                      **b** 0.057                      **c** 369 000                      **d** 850.9
- 4** Write the basic numeral for each of the following.
- a**  $7.5 \times 10^3$                       **b**  $1.07 \times 10^{-3}$                       **c**  $4.56 \times 10^{-1}$
- 5** Write the following correct to the number of significant figures indicated in the brackets.
- a** 8.916 (2)                      **b** 0.0589 (2)                      **c** 809 (1)
- 6** Write the following correct to the number of decimal places indicated in the brackets.
- a** 7.145 (2)                      **b** 598.241 (1)                      **c** 4.0789 (3)
- 7** Convert the following measurements into the units given in brackets.
- a** 7.07 cm (mm)                      **b** 2170 m (km)                      **c**  $0.1 \text{ m}^2$  ( $\text{cm}^2$ )  
**d**  $2.5 \text{ km}^2$  ( $\text{m}^2$ )                      **e**  $0.0005 \text{ m}^2$  ( $\text{cm}^2$ )                      **f**  $0.000 53 \text{ cm}^3$  ( $\text{mm}^3$ )  
**g** 5.8 kg (mg)                      **h** 0.07 L (mL)
- 8** Express the following percentages as decimals.
- a** 75%                      **b** 40%                      **c** 27.5%
- 9** Express the following percentages as fractions, in their lowest terms.
- a** 10%                      **b** 20%                      **c** 22%
- 10** Evaluate the following.
- a** 30% of 80                      **b** 15% of \$70                      **c**  $12\frac{1}{2}\%$  of \$106

- 11** A new LED smart television was valued at \$1038. During a sale it was discounted by 5%.
- a** What was the amount of discount?
  - b** What was the sale price?
- 12** Tom's weekly wage of \$750 is increased by 15%. What is his new weekly wage?
- 13** A 15-year-old girl working at a local bakery is paid \$12.50 per hour. Her pay will increase to \$15 per hour when she turns 16. What will be the percentage increase to her pay (to the nearest per cent)?
- 14** A leather jacket is reduced from \$516 to \$278. Calculate the percentage discount (to the nearest per cent).
- 15** After dieting for three months, Melissa who weighed 78 kg lost 4 kg and Jody's weight dropped from 68 kg to 65 kg. Calculate the percentage weight loss, correct to two decimal places, for each girl.
- 16** True or false?
- a** The ratio 3 : 2 is the same as 2 : 3
  - b**  $1 : 5 = 3 : 12$
  - c** 20 cm : 1 m is written as 20 : 1 in simplest form
  - d**  $3 : 4 = 9 : 12$
- 17** If a sum of \$800 were to be shared among a group of people in the following ratios, how much would each person receive?
- a** 4 : 6
  - b** 1 : 4
  - c** 2 : 3 : 5
  - d** 2 : 2 : 4
- 18** A recipe for pizza dough requires 3 parts wholemeal flour for each 4 parts of plain flour. How many cups of wholemeal flour are needed if 24 cups of plain flour are used?
- 19** The scale on a map is 1 : 1000. Find the actual distance (in metres) between two markers if the distance between the two markers on a map is:
- a** 2.7 cm
  - b** 140 mm
- 20** If 5 kilograms of mincemeat costs \$50, how much does 2 kilograms of mincemeat cost?
- 21** A truck uses 12 litres of petrol to travel 86 kilometres. How far will it travel on 42 litres of petrol?
-  **22** A earthquake measured 6 on the MMS. How many times stronger is a magnitude 6 earthquake compared to a magnitude 3 earthquake?