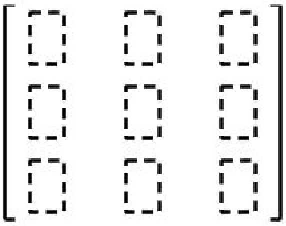
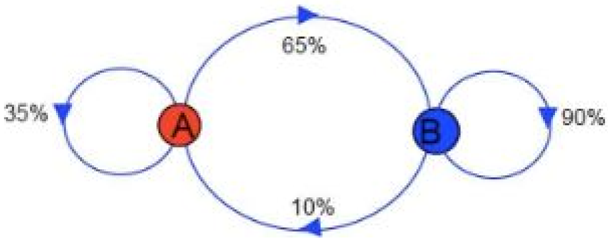
Concept –Transition Matrices

A transition matrix is a square matrix which describes the movement between a number of states. When setting up a transition matrix:



When being asked to make a transition matrix you are either provided with a written description of the situation or a transition diagram like the one shown below.



Once we know how the states change (______________________________) we need information about how we start in order to perform calculations.

The initial state is represented by a column matrix and is often labelled as S_o.

If in this situation A starts out at 90 and B starts with 30, S_0 will be

To find following states we can use the recurrence relation:

Initial State = S_0
State after 1 stage = S_1= _____________________

State after 2 stages = ___ = __________________ and so on.

We can represent this as a recurrence relation which will have the form $S_{n+1} = T \times S_n$

To do this with CAS

1. On a  _________________ page press  to open the templates.

2. Fill in the transition matrix then save it as T by pressing ________ → ______________

3. Premultiply the initial state matrix (S_0) by the saved transition matrix, this gives S_1.

4. Below type _____________ and press enter. This give S_2, continue pressing enter to get the next state.

This method of finding states is fine for small numbers but can be time-consuming when the state number gets bigger and bigger. To find one particular state we use the equation

$$S_n = T^n \times S_0$$

Example: A railway knows that 250 goods wagons will be needed to carry goods from Yarraville to Zagreb. At the end of each week, it finds that 10% of the wagons that started the week at Yarraville ended at Zagreb, and 8% of the wagons that started at Zagreb ended at Yarraville.

How many wagons will be at Yarraville and Zagreb at the end of 6 weeks, if 100 wagons started at Yarraville?

Step 1: Represent the situation with a transition diagram and/or a transition matrix.

Step 2: Identify the initial state.

Step 3: Substitute values into the formula $S_n = T^n \times S_0$

Step 4: Solve

Step 5: Respond to the question.

After a period of time, these types of situations will settle down into an equilibrium. In the example above this will mean that the number of wagons at each place will not change week to week.
This is known as the ___________________________

To find a steady state we have to use a large number for n. Typically n = ___________

To show that this is the steady state we need to demonstrate that it doesn't change. We

do this by performing the calculation again with n=________________

Example: What is the steady state of wagons at Yarraville and Zagreb?

Year 12 Past Exam 1 Question

Each week, the 300 students at a primary school choose art (A), music (M) or sport (S) as an afternoon activity.

The transition matrix below shows how the students' choices change from week to week.

$$T = \begin{array}{c} \text{this week} \\ \begin{array}{ccc} A & M & S \end{array} \\ \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.3 & 0.4 & 0.4 \\ 0.2 & 0.2 & 0.5 \end{bmatrix} \end{array} \begin{array}{l} A \\ M \quad \text{next week} \\ S \end{array}$$

Based on the information above, it can be concluded that, in the long term

A. no student will choose sport.

B. all students will choose to stay in the same activity each week.

C. all students will have chosen to change their activity at least once.

D. more students will choose to do music than sport.

E. the number of students choosing to do art and music will be the same.

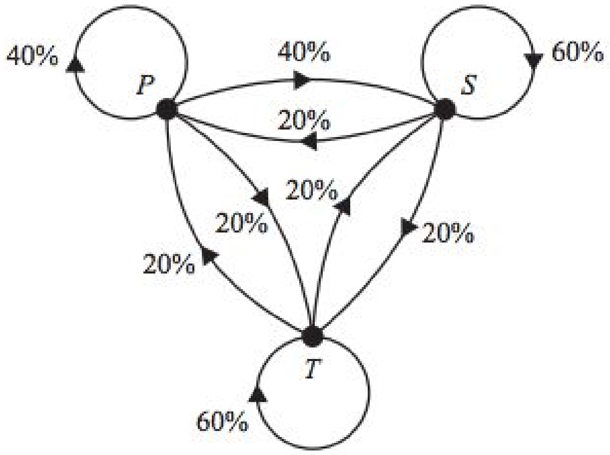
Year 12 Past Exam 2 Question

Question 2 (3 marks)

Junior students at this school must choose one elective activity in each of the four terms in 2018.

Students can choose from the areas of performance (P), sport (S) and technology (T).

The transition diagram below shows the way in which junior students are expected to change their choice of elective activity from term to term.



a. Of the junior students who choose performance (P) in one term, what percentage are expected to choose sport (S) the next term? 1 mark

Draw a transition matrix for the situation shown in the diagram above.

Worked Example

Choose either a transition diagram or a written description of a situation and use it to make a transition matrix. Use this to find the steady state for this situation.